9.5b Alternating Series, Checking Endpoints

Alternating Series Test with remainder

stop here

$a_1 - a_2 + a_3 - a_4 \dots a_n \} +$ remainder (error)

if
1. signs alternate
2. $a_{n+1} < a_n$
3. $a_n \to 0$

then series converges

remainder (error) $< a_{n+1}$

If a_{n+1} has a neg sign, overestimate

If a_{n+1} has a pos sign, underestimate

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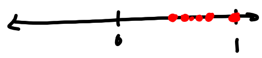
Prove the alternating harmonic series is convergent but not absolutely convergent



$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5} \dots$

1. signs alternate
2. magnitude gets smaller
3. seq. $\to 0$

series converges by AST

$\sum_{n=1}^{\infty} |a_n| = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{5} \dots$ diverges

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Conditional Convergence

$\sum (-1)^{n+1} a_n$ converges

but $\sum \left|(-1)^{n+1} a_n\right| = \sum a_n$ diverges

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4} \dots$

$\left(1+\frac{1}{3}+\frac{1}{5} \dots\right) - \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6} \dots\right)$

$\infty - \infty$

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Find the interval of convergence for the following series. Be sure to check the endpoints.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{2n}$$

$\frac{a_{n+1}}{a_n}$ $\quad L = \lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1+1}} x^{2(n+1)}}{2(n+1)} \cdot \frac{2n}{x^{2n} \cancel{(-1)^{n+1}}} \right|$

$\lim_{n\to\infty} \left| \frac{x^{2n+2}}{2n+2} \cdot \frac{2n}{x^{2n}} \right| = \lim_{n\to\infty} \left| x^2 \frac{2n}{2n+2} \right| = x^2$

$x^2 < 1$

$\sqrt{x^2} < \sqrt{1}$

$|x| < 1$

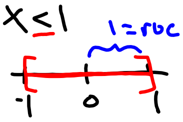
$-1 \leq x \leq 1$

$x = 1 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} \dots$

Conv. by AST

$x = -1 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{2n}}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} \dots$

conv by AST



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$$\sum_{n=0}^{\infty} \frac{(10x)^n}{n!}$$

find the ioc

$$L = \lim_{n\to\infty} \left| \frac{(10x)^{n+1}}{(n+1)!} \cdot \frac{n!}{(10x)^n} \right| = \lim_{n\to\infty} \left| \frac{10x}{n+1} \right| = 0$$

converges for all values of x

ioc $(-\infty, \infty)$

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$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{2n}$$

$$\lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{2n+2} \cdot \frac{2n}{(x-3)^n} \right| = \lim_{n\to\infty} \left| (x-3) \frac{2n}{2n+2} \right|$$

$L = |x-3| < 1$

$-1 < x-3 < 1$

$2 \leq x < 4$

$x=4$ $\sum \frac{1^n}{2n} = \frac{1}{2}\sum\frac{1}{n}$ harmonic div

$x=2$ $\sum \frac{(-1)^n}{2n}$ conv by AST

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